Chapter 1

Foam as granular matter

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What has foam in common with granular matter? What can these two active research fields learn from each other, where do they overlap? We approach these questions by reviewing a wide range of foam theory (mostly simulation) and experiment, set in the context of granular matter research.

1. Introduction

1.1. *History of foam research*

Foam and soap film research goes back to the dawn of modern science (Leonardo da Vinci,[25] Robert Boyle[12]). In 1873 the blind Belgian scientist Joseph Plateau published his masterful account of his own researches and the subject's previous history.[73] Many of those older references are also to be found in the classic work of Mysels *et al.* on soap films.[71]

Later Lord Kelvin took an interest in the subject.[94] The almost-simultaneous preoccupation of Kelvin with foams and Reynolds with granular media[74] drew motivation from the same source: the structure of the ether. This was the all-pervading medium that 19th century physics required for the propagation of light waves. Remarkably, the same question re-emerges today in modern form (the structure of space-time on the Planck scale) and theorists in that subject use granular and foam language interchangeably!

Foam structure, as first elucidated by Plateau, may be visualized/analysed from different perspectives, depending to some extent on the

Fig. 1. *From left to right*: 1D foam consisting of regularly spaced parallel soap films (bamboo foam);[22] 2D foam confined between two glass plates; 3D foam (the vertical gradient in liquid fraction is due to gravity-driven drainage).

wetness (liquid volume fraction) :

- a packing of bubbles
- a tessellation of cells
- a partitioning of space by films
- a network of lines (Plateau borders)

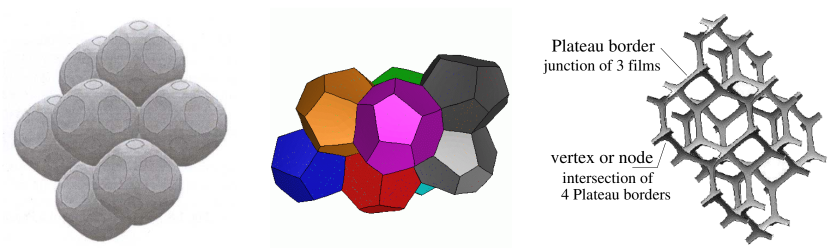


Fig. 2. Elements of foam structure: *from left to right,* packing of bubbles; partitioning of space by films: the Weaire-Phelan structure; network of Plateau borders.

It is simpler in two dimensions, as Cyril Stanley Smith explained,[84] but the two-dimensional (2D) system brings its own complications. For a start there are (at least) three varieties of ordinary 2D soap froths, as represented in Figure 3:

- Hele-Shaw cell : one layer of bubbles confined between two plates;[83]
- Plate/liquid: the bubbles float in liquid under a plate;[35,89]
- Free-floating bubbles.[14]

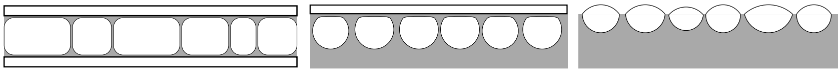


Fig. 3. Three types of 2D foams. *From left to right*: air bubbles confined between two glass plates; bubbles floating in liquid under a glass plate; monolayer of bubbles sitting at an air/liquid interface.

Note the contrast between the Bragg and Smith systems. Both were used to model grain growth in atomic crystals, but Bragg's monodisperse bubbles, small enough to be roughly spherical, represent individual atoms, whereas Smith's large polydisperse bubbles represent whole grains. Many foam experiments have been conducted with bubbles of diameters of a few millimetres (see 2.2). The effect of gravity is diminished for bubbles much smaller than the capillary length (about 1.6 mm for ordinary surfactants). Old ideas of how to make them monodisperse have been revived[88] and are of great interest in microfluidics.[30]

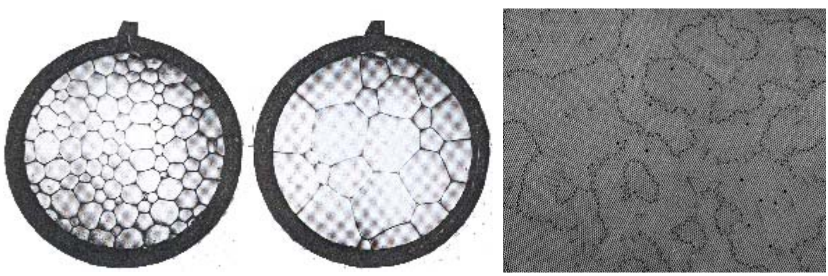


Fig. 4. *Left:* Smith's crystalline model[83] . *Right:* Bragg's bubble raft.[14]

1.2. *Space and time scales*

Three different length scales are pertinent in foam physics:

- the scale of the films where the physico-chemical properties of the stabilizing surfactants strongly influence the forces that determine dynamic properties of the foam;
- the scale of the individual bubbles, ruled by mechanical equilibirum of the films;

- the scale of the foam: smoothed continuum models can be built to model the foam as a non-newtonian fluid.[23,49]

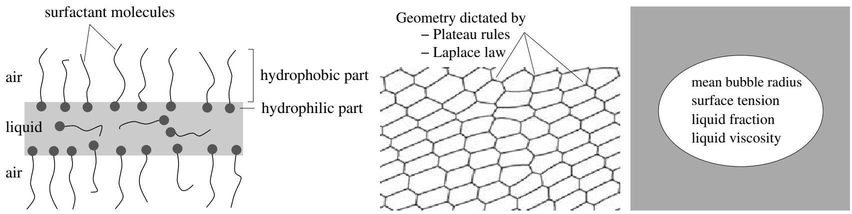


Fig. 5. The three length scales: *from left to right*, a film of surfactant solution, an ensemble of bubbles, foam as a continuous medium.

Most phenomena take place on very different time scales as well. Typically they are as follows:

- structural relaxations: a fraction of a second,
- drainage to equilibrium under gravity: a few minutes,
- coarsening, due to diffusion of gas: many minutes up to several hours.

These different timescales are very convenient in separating effects in experiments. The properties of general interest include those that are essentially static or can be described quasi-statically, like structure, stability, elasticity, coarsening, quasistatic rheology, light scattering, electrical and thermal resistance. Increasingly, properties that are truly dynamic are addressed, including details of transformations and structural relaxation, rate-dependent rheology, drainage, convective instability, size segregation. The stability of a foam in relation to film rupture depends crucially on the chemistry of the foaming solutions. Common detergent foams are remarkably stable against film rupture, over periods of days, and are ideal for the study of many generic foam properties.

Theory is sometimes complicated by the history-dependence of foam structure and hence its properties. Coarsening due to the diffusion of gas through the liquid cell walls eventually results in a foam whose average bubble diameter varies with the square root of time. However, the statistical properties of the bubble packing, such as the average number of faces per cell, remain the same.[39,85] The unique polydisperse system reached after a long period of coarsening (the scaling state) is thus a useful choice for a standard system, to avoid the arbitrariness of samples otherwise created.

Almost all of the above properties have closely equivalent counterparts for granular materials, so each field can be illuminated by the other. Is there a closer correspondence? Should we regard a gas bubble as a frictionless, compressible particle? Durian[33] and others have tried to integrate foam theory with granular and atomic systems, by using a single idealised model for all three, as we shall explain below. Do foams offer any advantages to those interested in exploring generic properties of disordered systems? There may be some. In particular, foams have relatively well-defined and understood local structure and interactions. There is no solid friction, although there may be equally problematic Marangoni effects. Laboratory equipment can be rudimentary: glass plates, tubing, simple air and water pumps. High speed video with sophisticated image analysis is becoming common, but direct measurements may still be performed to great effect. Besides, many foams are partially transparent, so that some limited observation of their interior is possible.

1.3. *Key physical parameters*

The key physical parameters describing an ordinary aqueous foam include those that characterize the liquid and its surface:

- surface tension: in practice, its value is usually less of that for pure water (surface tension $\gamma \simeq \frac{1}{3}\gamma_{\mathrm{water}} \simeq 24\mathrm{mN/m}$ is often a good estimate). Many expressions for pressures, forces, energies, elastic moduli, yield stress are proportional to γ. In simple theories this is often the only parameter of interest and hence tends to disappear entirely in simulations.
- bulk viscosity: it has been considered to control drainage and structural relaxation, but this is now questionable in some cases.
- surface properties, such as surface viscosity and elasticity, are of great importance but are difficult to capture in simple formulae. Their role has been studied by physical chemists for a long time but are only now being properly integrated into foam physics.
- a parameter that characterizes the permeability of films: it depends on the choice of gas and may be greatly reduced by an appropriate choice, in order to control coarsening by reducing the permeation of films.

Further basic parameters characterise the foam structure:

- the average bubble diameter d (or some other measure of size);
- the liquid fraction ϕ (often a function of position), which tends to zero in the limit of a dry foam;
- topological measures such as μ_2, the second moment of the distribution of the number of sides in a two-dimensional foam. This measures the spread of different types of cells (roughly corresponding to polydispersity), and crops up in various theories.

How can we access quantitatively these structural parameters in practice? We might squash and two-dimensionalise a three-dimensional (3D) foam sample between two plates, in order to estimate d. It is also possible to infere to some extent the size distribution in the bulk from what is observable on the sidewalls. To measure the liquid fraction ϕ, we may weigh a sample, or use light scattering, electrical resistance, gamma ray absorption or Archimedes Principle. As for granular media, the complete structure can also be accessed by tomography techniques (see section 5).

Remarkably, a lot of physics can be developed in terms of these few variables (leaving out surface viscosity and surface elasticity, if possible), after a few simplifying approximations. Foam physics is remarkably coherent and tractable at the level of ten percent accuracy, but becomes cluttered and obscured by a multitude of small effects if more precision is pursued. Most of the basic formulae that roughly capture physical properties can be found in the book of Weaire and Hutzler.[95] A fuller appreciation of the present breadth of the subject may be gathered from the proceedings of the last EUFOAM Conference in Potsdam,[66] or proceedings of previous European Foams Conferences.[92,101] The next is to be held at Nordwijk in 2008.

1.4. *Wet and dry foams*

The value of liquid fraction in a disordered foam can range between 0 (corresponding to an ideal polyhedral packing of bubbles) and approximately 0.36, where the latter value is the void fraction of a random packing of hard spheres (in two dimensions the equivalent value is approximately 0.16). If we call a foam *wet* if its (local) liquid fraction exceeds half of this value, the following estimate can be deduced for the thickness of the layer of wet foam lying on a pool of liquid,[95]

$$W_{\text{wet}} \simeq \frac{{l_0}^2}{d} \tag{1}$$

where l_0 is the capillary length, given by $l_0 = \sqrt{\frac{\gamma}{\rho g}} \sim 1.6$mm for usual surfactants (γ is the surface tension, ρ is the density of the liquid, g is the acceleration due to gravity). The number of bubble layers in a wet foam

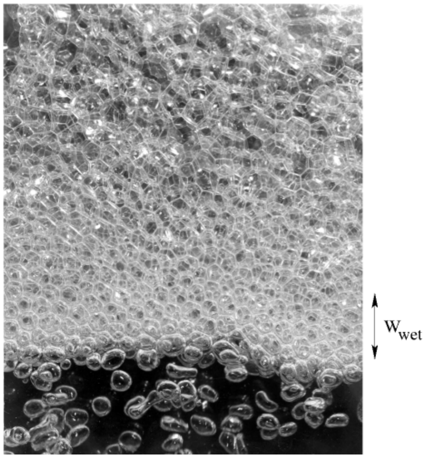


Fig. 6. An illustration of the gradient of liquid fraction. Here P is less than 5. (Photograph by J. Cilliers, Imperial College London).

under gravity is thus given by dividing W_{wet} by d and we shall define this here as the dimensionless *Princen number*

$$P = \left(\frac{l_0}{d}\right)^2 \tag{2}$$

in honour of the late Henry Princen, pioneer of foam drainage and rheology.

In the wet limit, a foam can be considered as a immersed granular material, with the differences that the grain weight is replaced by bubble buoyancy, and that the bubble surface is more deformable. On the other hand in the dry limit the foam can be seen as ai tessellation, rather like the Voronoi tessellation of a granular material.

1.5. *Emulsions*

Emulsions that have comparable structural scales to those of typical foams behave quite similarly, and may be termed bi-liquid foams. For static properties, replacing gas by liquid and bubbles by droplets changes little apart from the terminology. Of course, there are bound to be some important differences in dynamic properties, wherever the viscosity of the enclosed liquid is significant. This is noticible in particular in so-called forced-drainage experiments, where the continuous phase is continuously replenished to avoid a drying out of the foam or emulsion.[72] Whereas in foams the bubbles undergo convective rolls provided the rate of added liquid exceeds a certain threshold (cf. 2.3), in emulsions flow instabilities can take the form of density waves.[43]

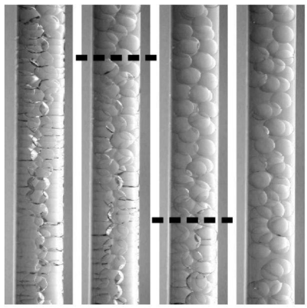


Fig. 7. Drainage wave passing through an emulsion of oil in water (the dashed lines indicate the position of the wave front at successive times).[43] In the case shown this results in a high water volume fraction($\phi \sim 0.5$) with agitated motion of the oil droplets.

2. Static properties

Different styles of modelling/simulation have been developed for static structures and quasistatic properties. They include

- detailed representation of the structure (films, Plateau borders, junctions);
- vertex models (not discussed here);
- soft sphere/disk models.

2.1. *Structure*

The first of the above models is a precise representation of foam structure, after it has been idealised, primarily by treating the films as infinitesimally thin.

In two dimensions this was accomplished for a dry foam by Weaire and Kermode in 1984.[97] Wet foam was similarly simulated by Bolton and Weaire[9,10] (cf Fig. 15). Nowadays the specifically developed codes have been largely replaced by the 2D version of the Surface Evolver of Ken Brakke.[15] Equilibration is usually straightforward, but some ques-

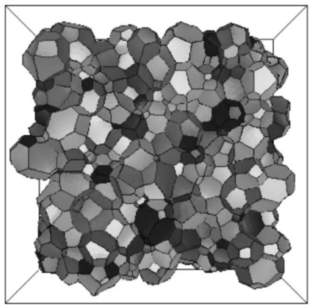


Fig. 8. A 3D foam generated with Surface Evolver.

tions arise. What starting configuration is to be used? Most simulations start from a rather arbitrary Voronoi partition, as this is convenient. Is the resulting structure unique (for a given final topology)? The answer to the second question appears to be no, in general, but yes in practice for typical, finite samples.

Brakke's Surface Evolver[15] also provides the standard procedure for equilibration in the case of 3D foams. Whereas the lines in 2D foams are always circular arcs, no such simplification is available in 3D, and a fine tessellation is needed to represent the surfaces. Helpful features of the Surface Evolver package include calculation of the Hessian matrix and its eigenvalues, which can help in characterising instabilities.[100]

The chief practitioner of the application of this methodology to foams has in recent years been Andy Kraynik. His extensive exploration of random equilibrium structures of monodisperse foams[54] has revealed a wide range of possibilities, echoing the story of the various mondisperse hard-sphere packings in the theory of granular materials. This has provided a vindication of old experimental work of Matzke,[65] who laboriously assembled foam samples by adding identical bubbles one at a time; he then analysed their contents with even greater labour. Some of us distrusted the detailed results of that unique effort, but Kraynik[54] has now confirmed them.

2.2. *Crystallization*

Early observations have been made by Bragg and others[13,14,47,59,60] of the crystalline ordering of a 2D raft made of small bubbles (see Fig. 4). This system has been used as an analogue to solid crystals and the interactions between bubbles have been modeled accordingly by a short-range repulsion (due to the bubble distortion) and a long-range attraction (due to the distortion of the water surface[90]).

Some recent developments have been made by extending Bragg's bubble raft concept into three dimensions. When the bubbles are typically smaller than 500μm, the Princen number becomes greater than 10, which allows equilibrium samples of wet 3D foams, with more than 10 layers. Remarkably, it has been observed that these small bubbles order spontaneously in three dimensions, with an apparent preference for FCC structure.[88] The existence of stacking faults, grain boundaries, dislocations, all phenomena already identified in 2D bubble crystals, is yet to be investigated in detail. This spontaneous ordering is not observed in emulsions and granular media.

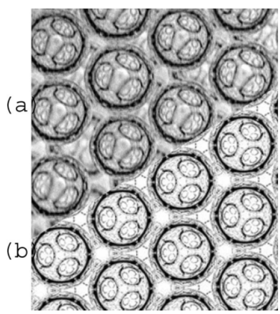


Fig. 9. Crystalline structure of bubbles in 3D: (a) A photograph of the surface of the foam. (b) A ray tracing simulation of a (111) fcc packing shows detailed agreement with the experiment.[88]

2.3. *Drainage*

Drainage is closely analogous to sedimentation of suspensions, gas transport in fluidised beds, and water transport in wet soils. The liquid therefore passes through an essentially static structure, in the manner of liquid transport in a porous medium, but with an important difference. The local liquid fraction is not fixed but is to be determined to be consistent with liquid pressures: the foam *breathes* liquid in and out, with a change in Plateau border cross-sectional area. A simple dependence of rate of uniform flow (under gravity) on liquid fraction is often found experimentally: the flowrate Q goes as ϕ^2. This is rationalised in terms of Poiseuille flow, a consequence of high surface viscosity, so that the Plateau borders act as channels with a no-flow condition at their surface. Some surfactants produce a low surface viscosity instead, and a different theory is needed.[52,53,91,94]

The non-uniform flow through the foam can be described by the partial differential equation

$$\frac{\partial \phi}{\partial \tau} = \frac{\partial}{\partial \xi}\left(\frac{1}{2k+1}\frac{\partial \phi^{k+1/2}}{\partial \xi} - \phi^{k+1}\right) \tag{3}$$

where the parameter k characterizes the type of flow ($\phi = 1$ for Poiseuille flow and $\phi = 1/2$ for plug flow). Under forced drainage, the wetting front takes the form of a solitary wave.[98] There is some connection with wetting fronts in soil mechanics, a branch of granular materials. In that field also, solitary waves describe wetting and are the solutions of kinematic equations, of a somewhat more complicated and uncertain form.

Furthermore a convective instability appears at high enough flowrate.[45] This seems roughly similar to instabilities in fluidized beds, but has a simpler form. It can also be compared to the convection rolls that appear in a shaken granular media above a certain threshold of acceleration.[75] In the unstable regime there is size segregation in polydisperse foams, with smaller bubbles tending to collect at the bottom.[46] This seems closely analogous to similar effects in granular materials.[18] The fact that the convective instability occurs at a lower flow rate when the drainage column is tilted[24] can be seen as analogue to the Boycott effect in sedimentation of suspensions.[11,31]

3. Dynamic properties

3.1. *Rheology*

Continuum model

As in the case of drainage, rheology has been treated with continuum models (see e.g[64,93]), most recently to analyse 2D shear experiments (see 3.2 and[23,49]). Instead of a detailed simulation, a coarse-grained description proceeds in terms of fields that represent the local average of liquid fraction, velocity, etc. Foam is a solid with well-defined elastic properties for low stress, but plastic at higher stresses, and it flows indefinitely above a certain yield stress. This central property is shared with granular material. Unlike the latter, foam has a very large elastic/plastic regime in terms of the strain range that may be explored before the yield stress is reached. There are simple formulae for elastic modulus and yield stress.

In common with granular material, avalanches of topological changes can

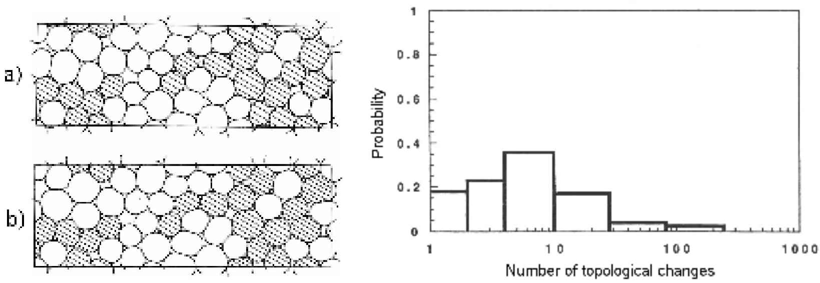


Fig. 10. Early data[44] showing avalanches of bubble rearrangements in wet foam due to applied shear. *Left*: Bubbles that change their nearest neighbour relationship are shaded. *Right:* The probability distribution of topological changes displays a long tail.

be observed (see Fig. 10), particularly in wet foams.[44] There should also be force chains in the wet limit: they have been observed in simulations by Durian,[34] but a method of detecting these experimentally has not been devised.

Above the yield stress, i.e. once the foam flows, strain-rate dependent effects must be taken into account. They have proved difficult to capture convincingly in theory. The Bingham model is a useful heuristic device, in which the yield stress is added to a term proportional to the strain rate, as in a newtonian liquid. The effective viscosity can then be expressed as

$$\eta_{\text{eff}} = \frac{\sigma}{\dot{\epsilon}} + \eta_p \tag{4}$$

where σ is the shear stress, $\dot{\epsilon}$ the strain rate and η_p the plastic yield.

In reality, the introduction of a nonlinear viscous term (the Herschel-Bulkley model) seems necessary, but its validity and precise origins need to be explored. Note also that the Bingham model (eq. 4) applies only to shear strain that varies monotonically.

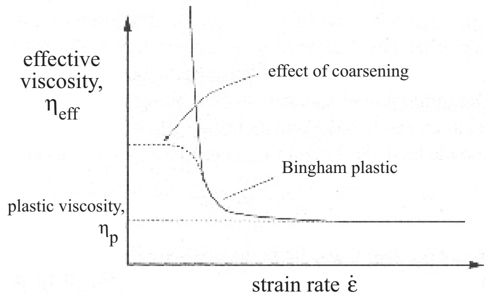
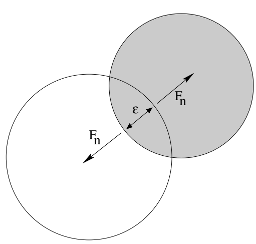


Fig. 11. Variation of effective viscosity with strain rate.

Soft-disk model

In the case of a wet foam, another approach at the bubble scale can be useful. Douglas Durian investigated the rheological behaviour of a foam made of spherical bubblest by modelling their interaction with a heuristic model:[33,34] the bubbles remain spherical but can overlap and repell each other elastically. This soft disk or sphere model is also the base of molecular dynamics simulations in granular materials and despite its simplicity has proven very efficient in reproducing the main feature of dry granular flows. However, the validity of this simple interaction model can be questioned.



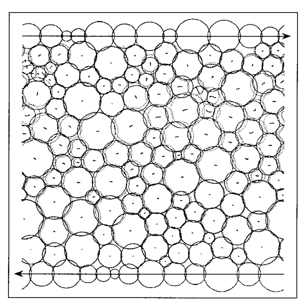


Fig. 12. *Left:* Elastic interaction between overlapping bubbles. *Right:* Topological rearrangement in a 2D foam under shear (reprinted figure with permission from.[34] Copyright (1997) by the American Physical Society).

Theoretical calculations applied to special cases[55,62,70] show that the true interactions are neither quadratic nor additive, even in the limit in which the bubbles are barely touching. In other words, the bubble-bubble potential depends on the confinement, and in particular the number of contacts of each involved bubble. More general cases (in which the environment of each bubble is not symmetrical) raise even greater problems, in any attempt to put soft-sphere models on a firm foundation for foams.

Dissipation

Dynamic properties require models that include dissipation, and the work of Durian brings these in straightforwardly by adding a viscous term to the spring force, which can only be accepted as a heuristic device. The true rate-dependent forces needed are very much a matter of debate, and include both hydrodynamic contributions that do not scale in a simple way (energy is dissipated due to shear flow of the viscous liquid within the soap films and Plateau borders), and surface contributions.[17]

In 2D there is another viscous force, of great importance. At least whenever there is at least one plate involved, wall drag is very important, and introduces effects not present in 3D foam, so the 2D model system must be an unreliable guide to its dynamic properties.

What is the nature of this force ? This is the Bretherton problem,[16] which has been subject to numerous investigations, experimental,[20,87] theoretical[28] and numerical[80] (see also Fig. 13. Various power laws have been derived, rather than a force which is simply linear in the bubble velocity relative to the wall. This nonlinearity arises from the dependence of the local structure (in particular, the thickness of the film at the wall) on velocity.

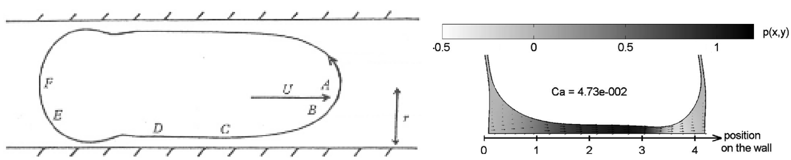


Fig. 13. *Left*: Section of a bubble in a tube as modeled by Bretherton.[16] *Right*: Deformation of the bubble and corresponding flow from numerical simulations.[80]

3.2. *Shear banding*

Since the challenges of rheology and flow are so intractable, it is natural to have recourse to 2D model systems as a starting point. In the case of foams this has been the strategy of (among others) the groups of Glazier,[1] Graner[29] and Cantat.[19]

Amongst the main features, the phenomenon of shear-banding in sheared granular materials has been widely investigated.[37,38,42,48,61] 2D foam rheometers have been constructed by the groups of Debregeas[27] and Dennin.[57] In the first version, foam is sheared in a Couette (cylindrical) geometry (Fig. 14). It shows strong shear banding, localised at the inner boundary, with a velocity distribution decaying exponentially. The existence of a similar phenomenon in 3D is still subject to debate. Remaining in 2D, several explanations have been proposed. An early and initially convincing theory was based on a quasistatic description and detailed simulation;[50] it is difficult to summarise, and remains problematical in several respects.

A second and entirely different approach based on a continuum model

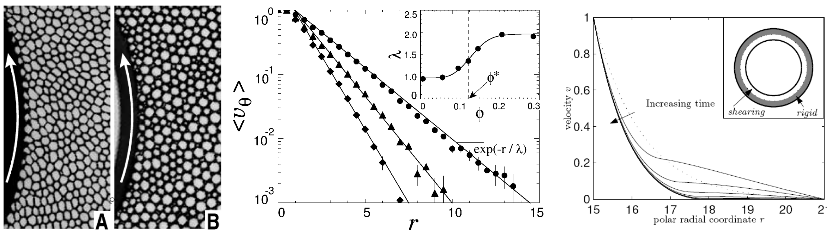


Fig. 14. *From left to right:* Experimental Couette setup for dry and wet foams; Exponential decrease of the tangential velocity across the gap (reprinted figures with permission from.[27] Copyright (2001) by the American Physical Society); Prediction of the velocity profile by a continuum model.[23]

leads naturally to the shear banding in agreement with qualitative observation,[23,49] and to several further results that seem to validate the model and have predictive power.

Not only is this an essentially dynamic model, but it includes as its crucial ingredient the wall drag force which we have already pointed to as important in 2D. Apart from that the foam is modeled with the elementary Bingham model defined in section 3.1. The localisation length is determined solely by the viscous coefficient, together with the coefficient of wall drag, whereas neither coefficient appears in the previous theory.

All is not clear yet, but we seem to be on the road to a full understanding. The comparison with granular experiments is intriguing (see the work of Behringer in this volume). In the latter case, much is similar. While a wealth of fine detail has been observed and analysed but the broader questions remain. Could the continuum model be relevant here? Experimental tests might help to resolve that question.

3.3. *Dilatancy*

A recurrent theme in the description of granular materials is dilatancy, but it is often as vague as in Reynolds' original and confusing accounts of it.[74] It requires some clarification in seeking its analogue in foams. Very loosely, we may say that dilatancy is the tendency to expand when a material is sheared. But shearing may be *static* (elastic or partly plastic) or *dynamic*, above the yield stress. It would seem that we must at least distinguish elastic and dynamic dilatancy.

In foams there is a quite well defined range of elastic distortion which can be well described by simulation and theory. It therefore offers the opportunity for an analysis of elastic dilatancy which presents little uncertainty. This was undertaken by Weaire and Hutzler,[77,96] using 2D simulations. It should be noted that elastic dilatancy fits within the classical theory of elasticity, being merely a third-order effect; in general it is of arbitrary sign. Indeed it was found to be negative in the theory for dry 2D foams. The analogy with granular materials is to be found in wet foams: the effect rapidly becomes positive as liquid fraction increases. We may quantify the effect in a manner that is suggestive of possible experiments by asking: what is the difference of liquid fraction between two samples of which one is under static elastic shear, the other not, with contact between the two? The answer given was that the difference has a maximum of a few percent,

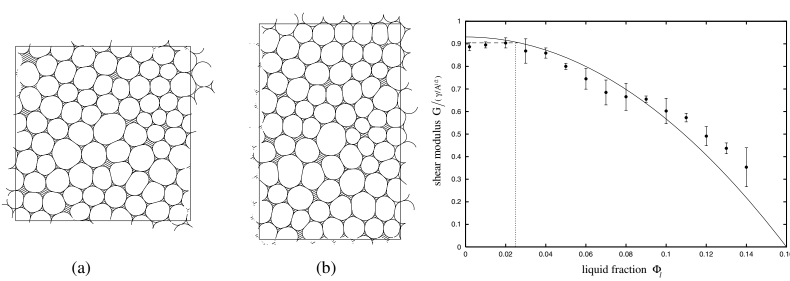


Fig. 15. *From left to right:* Computer simulations of a two-dimensional foam with liquid fraction $\phi = 0.07$ using the software PLAT:[9] (a) unstrained; (b) under extensional strain $\epsilon = 0.23$; Variation of shear modulus with liquid fraction.[96]

halfway towards the wet limit (i.e. a liquid fraction of fifteen to twenty percent). Gravity complicates experimental design and the prediction is not as yet confirmed by measurement.

Nothing has been predicted for dynamic dilatancy but it also seems a good candidate for measurement. In the case of granular materials, it is to be associated with the name of Bagnold,[6] since his measurements were indeed for the dynamic case, and even produced a variation of dilatancy with shear rate. However, a recent critical review of this is skeptical.[41] In parallel with other attempts to make sense of foams above the yield stress, this presents another opportunity for experiment and for comparison with granular materials.

4. Bubbles as soft grains ?

Recently, Vanderwalle *et al.*[7,21] revived the analogy between wet foams and granular materials by investigating bubble flows in the classical configuration of a 2D flow through a narrow aperture.[8,40,67] To maintain a high liquid fraction in the foam it is necessary to avoid drainage and thus to keep the effective gravity low enough by confining the bubbles under a slightly inclined plate (Fig. 16). Within this setup, the small bubbles remain roughly spherical and move independently, thus behaving like deformable grains. Whereas in the case of granular media the grains undergo

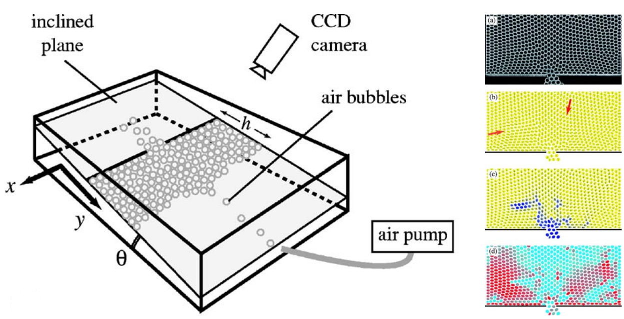


Fig. 16. Experimental observation of the flow of bubbles through a narrow aperture (reprinted figures with permission from.[7] Copyright (2006) by the American Physical Society).

solid friction and can therefore form contact arches, the bubbles experience only viscous forces. Thus Beverloo's law giving the flow rate Q as a function of the particle size d and the aperture D reads differently:[7]

$$Q \propto \begin{cases} g^{1/2}\left(D - kd\right)^{1/2} & \text{for solid grains} \\ (g\sin\theta)^{3/2}\left(\frac{D}{d} - k\right)^{1/2} & \text{for bubbles} \end{cases} \quad (5)$$

In Trinity College Dublin, another classical experiment for granular media has been reproduced with small bubbles: a monolayer of small bubbles is confined under a tilted rotating plate, thus forming a 2D rotating tumbler. This type of flow can be simulated by adapting Durian's soft disk

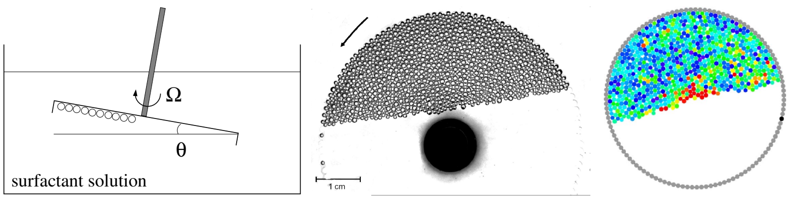


Fig. 17. Bubbles in a rotating drum: experimental setup, snapshot of the bubbles during rotation, and numerical simulation.[36]

model and incorporating bubble inertia and wall friction. The viscous dissipation experienced by a single spherical bubble sliding along a plate has been investigated by Aussillous and Quéré,[5] who identified both Stokes and Bretherton components:

$$F_d = a\eta V r + b\kappa\gamma^{1/3}(\eta V)^{2/3}r^2 \quad (6)$$

where η is the dynamic viscosity of the bulk fluid, V the velocity of the bubble, κ^{-1} the capillary length and r the bubble radius. An important question is whether this formula, in particular the term for bulk friction, remains valid in the case of a system of bubbles.

5. Seeing inside foams (Computed Tomography)

The dynamics and evolution of 2D foam structure are generally understood[32,86,99] since the foam structure can be directly seen and dynamical processes (such as T1 and T2 transformations) are easier than the three dimensional ones to visualize and understand. In 3D, however, the challenge is to see and characterize the full inner structure of the foams which is not immediately visible from outside. Many techniques, primarily based on direct imaging, have been developed over the past few decades to probe the internal microstructure of complex materials.[51,79] Direct measurement of the 3D structure of porous materials is now readily available from synchrotron and X-ray Computed Tomography (CT). These techniques provide the opportunity to measure experimentally the complex morphology of the microstructure of materials in three dimensions, in a non-invasive way, at resolutions down to a micron. One can then base calculations directly on the measured three-dimensional microstructure.[2,3,26,63]

Granular materials

In recent years, X-ray CT has been employed in the field of granular materials to characterize the granular structure of single-sized hard spherical beads.[2,3,76,81,82] This has allowed the researchers, for the first time, to investigate the static geometry of large packings of up to 150, 000 monosized hard spheres (See Fig. 18 and[2]). Attempts are under way to study the dynamics of the compaction process of elastic and deformable beads in three-dimensions, a close analog to 3D wet foams, using X-ray tomography (see the paper of Saadatfar and others in this volume). This might shed light onto the understanding of topological changes of amorphous microstructured materials responding to applied stress, which bears close analogy with the deformation of 3D liquid foams (Fig. 18).

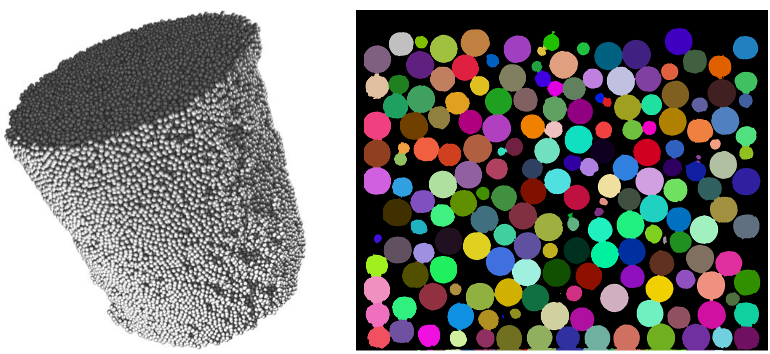


Fig. 18. *Left:* Reconstruction of a packing of 150,000 hard spheres obtained from tomography.[2] *Right:* A 2D slice through the tomogram of a 3D packing of deformable rubber balls.

Cellular Solids

The physical and mechanical properties of cellular solids are a direct consequence of their complex microstructure. Linking properties to structure will lead to an understanding of how cellular solids can be optimised for given applications. This goal can be achieved by utilizing tomography to acquire the density map of the specimens (See Fig. 19 and[78]). Indeed, the manufacturing process of both open and closed cell foams are very similar to the packing of spheres in 3D as can be immediately deduced from figure 19.

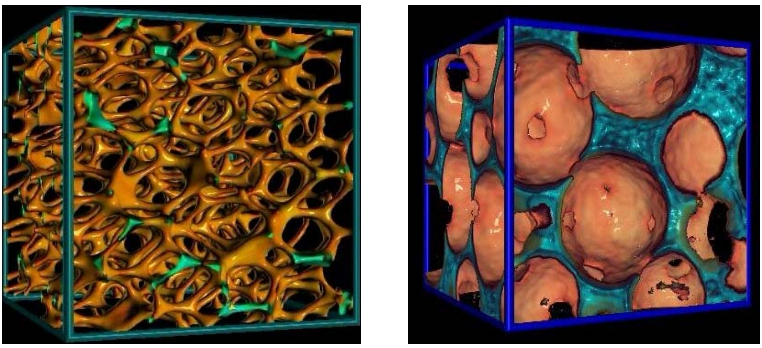


Fig. 19. Images of open-cell aluminium foam (left) and close-cell polyurethane foam (right) obtained by tomography.[78]

Aqueous foams

In the realm of aqueous foams, the first experimental investigation of a 3D analog of von Neumann's law was carried out by means of optical tomography.[68] The purpose of this experiment was to study foam structure and dynamics simultaneously by investigating morphology, topology, and dynamics of a 3D foam. Most recently X-ray tomography was used to study the evolution of initially 7000 bubbles in foams with liquid fraction ϕ between 0.1 and 0.2 percent.[56] The main aim of such work is the determination of a growth law for bubbles, i.e. how/whether the number of faces of a bubble determines whether the bubble will shrink or grow during coarsening.

6. Conclusions

Drawing an analogy between foams and granular materials is certainly tempting and rewarding, and currently also en vogue, as exemplified by the publication of popular science books[4,69] combining both themes (a collection of original articles on granular systems, foams, emulsions and supensions can be found in[58]).

However, care needs to be taken when trying to pursue this analogy in great detail. The interactions between soft bubbles and hard grains are different, and in both cases not yet sufficiently understood. Idealised simulations and toy models have proved instructive, but the time has come to rebuild the foundations of the subject on more solid grounds. This is especially the case whenever dynamic effects are considered.

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